A Tree Model for Pricing Convertible Bonds under different call policies and conversion scenarios.

Adviser： Prof. Tian-Shyr, Dai

Speaker：Hao-Han Chang

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  - How about obtaining the conversion shares.
- Backward Induction
- Model restrictions
- Numerical method example

Motivation and Literature Review

- What is Convertible bond?
  - A convertible bond is a hybrid security with debt and equity-like features

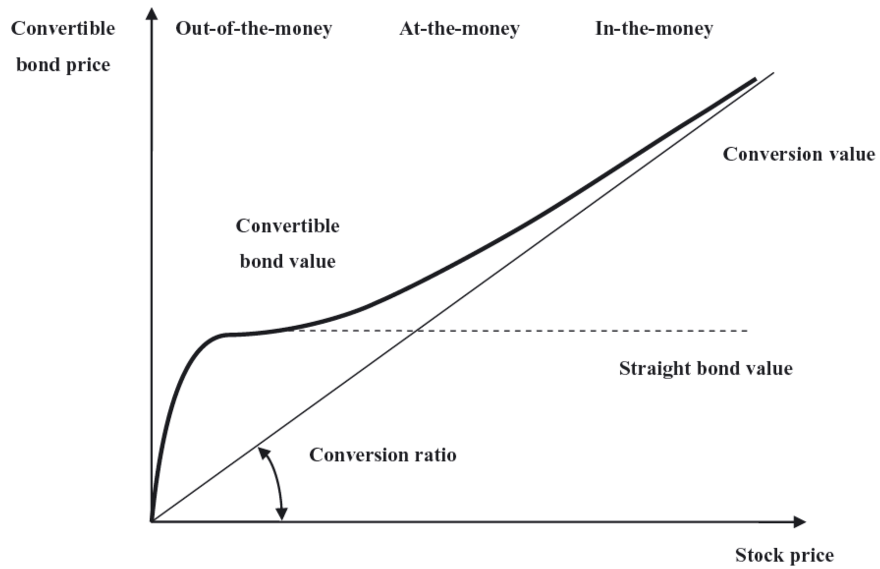


Figure from JA. Batten and KL. Khaw and M R. Young (2013)

  - In this proposal only talks conventional convertible bond, the payoff depends on the underlying firm stock.

Motivation and Literature Review

- **How to price convertible bond?(1/4)**
  - Since the early work of **Ingersoll (1977a)** and **Brennan and Schwartz (1977)**, an assortment of arguments, model specifications, and solutions to the pricing models have been developed to reflect real market specifications in the pricing of convertible bonds.

$$Convertible\ bond = Straight\ bond + Option$$

  - **Ingersoll (1977a)** if perfect market assumptions hold, with no dividends and constant conversion terms, it is never optimal for investors to opt for early conversion. Investors will only convert involuntarily at maturity, or at call, if the conversion value exceeds the call price, or the face value.
  - **Brennan and Schwartz (1977)** if dividends are paid and adverse changes in the conversion terms are not allowed, the optimal conversion strategy is to convert a convertible bond either immediately prior to a dividend payment date or at the maturity.

Motivation and Literature Review

- **How to price convertible bond?(2/4)**
  - **Call issue**
    - **Brennan and Schwartz(1977,1980)** Convertible corporate Bonds should be immediately redeemed if their conversion value is greater than the Call Price.
    - **Mikkelson (1985)** in the article statistics, the company average to convertible bonds convertible prices higher than the redemption price of 43.9% will be redeemed.
    - **Sarkar(2003)** predicts that early calls will be associated with high coupon and low call premium, dividend income, asset volatility, tax rate and interest rate; and late calls will be associated with high call premium, dividend income, tax rate and interest rate, low coupon and asset volatility.
    - **Chen, Dai and Wan (2013)** shows that credit risk and tax benefit have considerable impacts on the optimal strategies of both parties. The shareholder may issue a call when the debt is in-the-money or out-of-the-money. This is consistent with the empirical findings of "late and early calls."

Motivation and Literature Review

- The Table shows the annual conversion volume for the nine year 6.5% convertible bond issued by KSB in 1983. A conversion was feasible during a time period from 1984/1/1 until 1992/12/15.

year	83	84	85	86	87	88	89	90	91	92
conversion volume in %	–	70.43	1.23	0.07	0.51	15.06	3.03	1.45	1.31	6.19

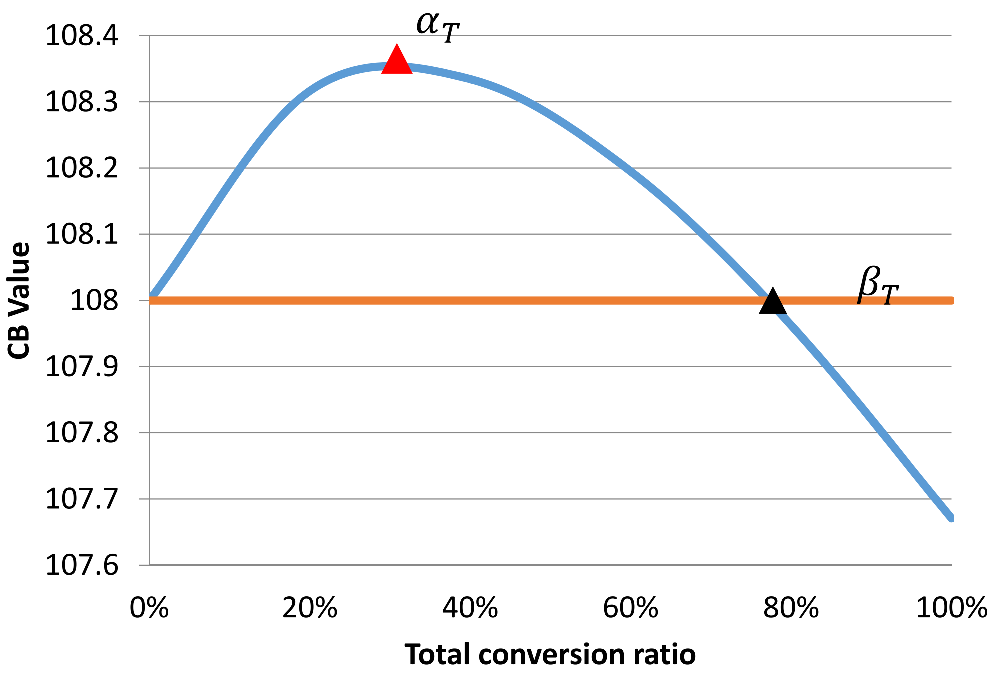
- Similar issues have also taken place in Taiwan's bond market

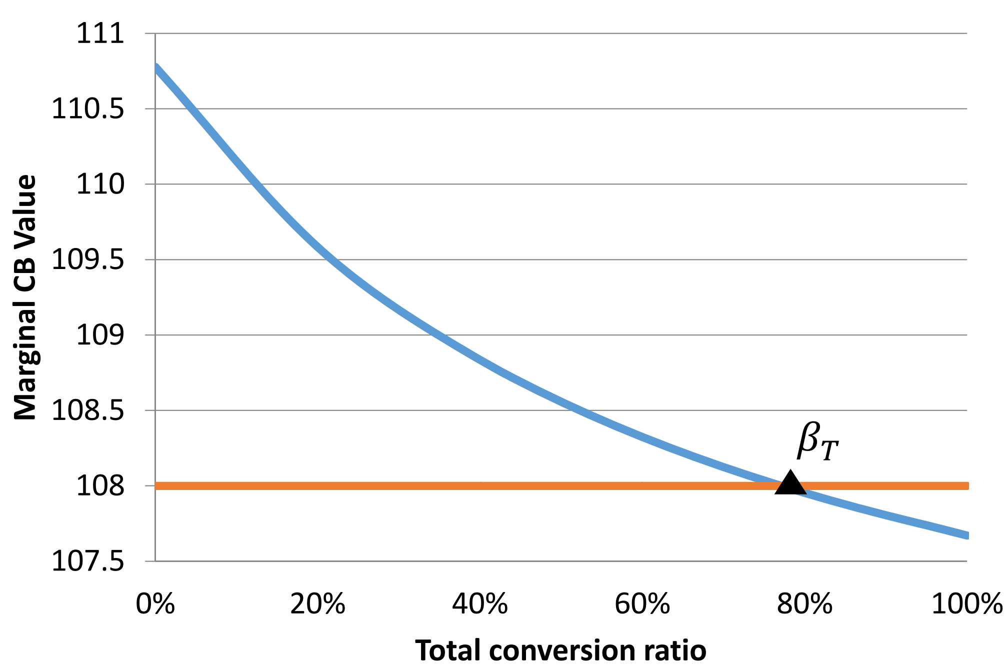
公司代碼	年月	到目前餘額(1000)	轉換價格	上市日	幣別	到期日
15043 東元三	2013/12	0	20.7	2012/07/12	NTD	2013/12/29
15043 東元三	2013/11	28,000	20.7	2012/07/12	NTD	2013/12/29
15043 東元三	2013/10	112,300	20.7	2012/07/12	NTD	2013/12/29
15043 東元三	2013/09	598,200	20.7	2012/07/12	NTD	2013/12/29
15043 東元三	2013/08	677,200	20.7	2012/07/12	NTD	2013/12/29
15043 東元三	2013/07	1,262,800	20.7	2012/07/12	NTD	2013/12/29
15043 東元三	2013/06	1,577,600	21.5	2012/07/12	NTD	2013/12/29
15043 東元三	2013/05	2,147,400	21.5	2012/07/12	NTD	2013/12/29
15043 東元三	2013/04	2,147,400	21.5	2012/07/12	NTD	2013/12/29
15043 東元三	2013/03	2,542,100	21.5	2012/07/12	NTD	2013/12/29
15043 東元三	2013/02	2,646,600	21.5	2012/07/12	NTD	2013/12/29
15043 東元三	2013/01	2,825,300	21.5	2012/07/12	NTD	2013/12/29
15043 東元三	2012/12	3,000,000	21.5	2012/07/12	NTD	2013/12/29

Motivation and Literature Review

- **How to price convertible bond?(3/4)**
  - **Conversion issue**
    - As noted earlier, all these approaches implicitly or explicitly assume that all convertible bond holders completely convert their bonds at the same time or not at all. This behavior is denoted by *block conversion*.

    - **Constantinides (1984)** show that sequential exercise can be optimal if one investor holds all convertibles (the *monopolistic* case), or if *competitive* convertible bond holders act strategically (unrestricted conversion).
    - in the absence of straight bonds and call provision, the monopolist warrant holder exceeds the warrant price in the competitive or in the block warrant holder's equilibrium.





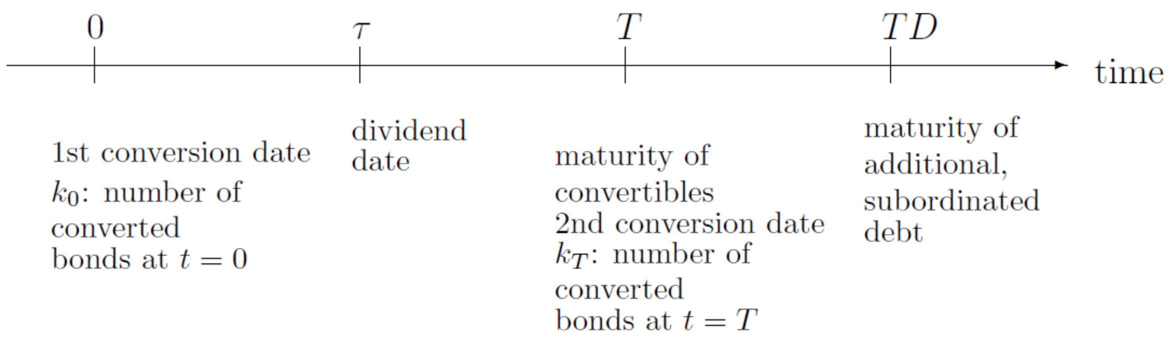
- The converted value
- Unconverted value

α_T : Optimum conversion ratio of Monopoly at maturity date

β_T : Optimum conversion ratio of Competitive at maturity date

Motivation and Literature Review

- **How to price convertible bond?(4/4)**
  - **Bühler, Wolfgang and Koziol, Christian(2002)** value convertible bonds and to characterize the optimal conversion strategies when a firm has additional debt in its capital structure. This paper focus on the differences between block conversion and unrestricted conversion. in general, a *sequential conversion* is optimal rather than a block conversion.



Motivation and Literature Review

<table>
<tr><th></th><th></th><th>Monopolistic</th><th>Competitive</th><th>Block</th></tr>
<tr><td rowspan="3">Call provision</td><td>Only one conversion date</td><td>–</td><td>–</td><td>Ingersoll (1977a)
Brennan and Schwartz (1977)</td></tr>
<tr><td>Two conversion dates</td><td>–</td><td colspan="2">Bühler, Wolfgang and Koziol, Christian(2002)</td></tr>
<tr><td rowspan="2">Conversion every time</td><td colspan="3">This proposal</td></tr>
<tr><td>Not call provision</td><td colspan="3">Constantinides (1984)</td></tr>
</table>

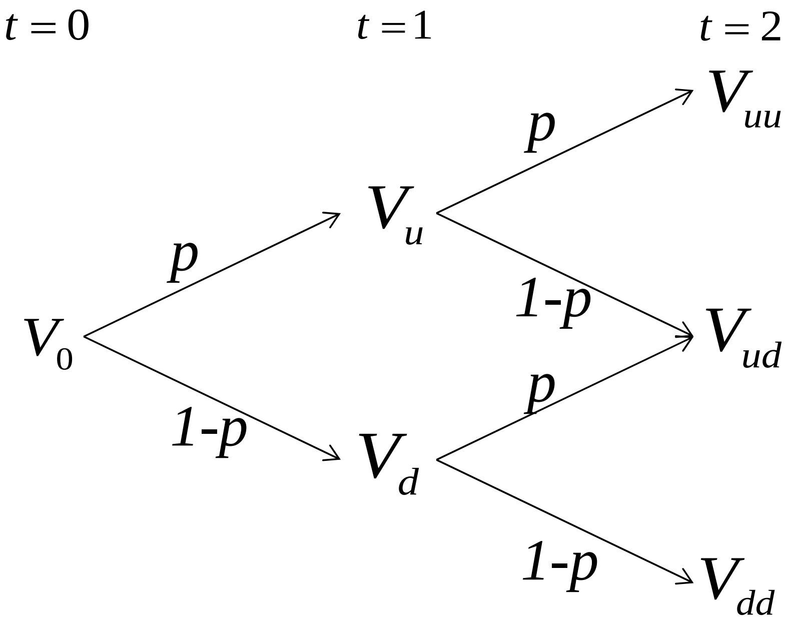
- *Galai and Schneller* (1978) one exercise date
- Emanuel (1983) and many exercise dates

Motivation and Purpose

- **Motivation**
  1. Base **Ingersoll (1977a)** "*If the perfect markets, no dividends, and constant conversion terms assumptions are valid, then a callable convertible will never be converted except at maturity or call*". However, voluntary conversion before maturity and sequential conversion are observable.
  2. Conversion policy can be complicated. They can be affect by :
     a. Convertible bonds may be held by one or more holder. *Emanuel (1983), Constantinides (1984) and Spatt and Sterbenz (1988)*
     b. The shares of the converted bonds may be come from issuing new equity or repurchasing outstanding from the market.
     c. Stochastic interest rate. *Brennan and Schwartz(1980)*
- **Purpose**
  1. Call policy and conversion policy may be not a one-way relationship.
  2. Try to find the correspondence between the call and conversion policy strategy.

Binomial tree pricing model and Backward Induction

- Cox, Ross and Rubinstein (1979)

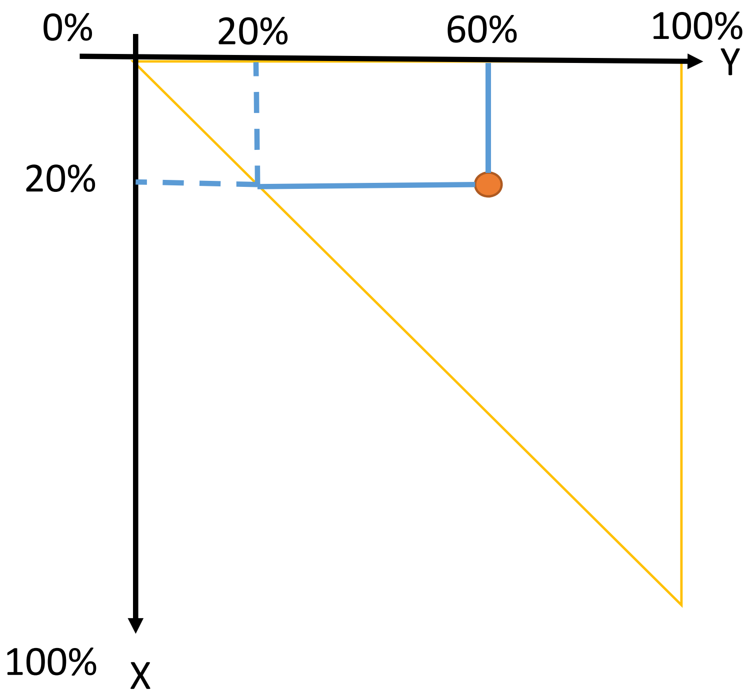


- Assume asset stochastic process

$$d\ln V(t) = \left(r_f - q - \frac{\sigma_v^2}{2}\right)dt + \sigma_v dZ_v$$

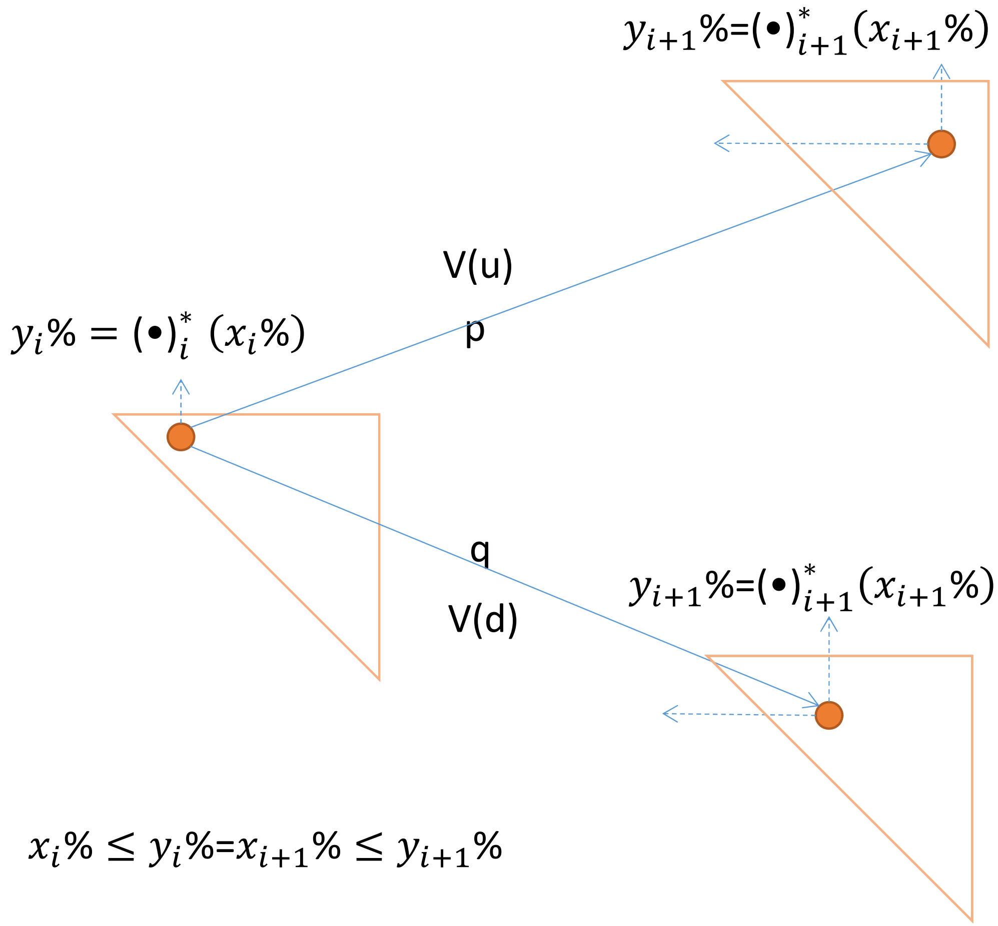
- Risk-neutral：
  up factor：$u = e^{\sigma_V\sqrt{\Delta t}}$ down factor ：$d = \frac{1}{u}$
  up probability：$p = \frac{e^{(r_f - q)\Delta t} - d}{u - d}$ down probability ： $1-p$

Sequential conversion



Y axis：The current cumulative conversion ratio

X axis：Pre-cumulative conversion ratio



$x_i\% \leq y_i\% = x_{i+1}\% \leq y_{i+1}\%$

(•) is optimum conversion ratio. According to Block, Monopolistic and Competitive case.

* is call, uncall, called.

Comparison of conversion scenarios

CBValue	The current cumulative conversion ratio (y%)						
Pre-cumulative conversion ratio (x%)		0%	20%	40%	60%	80%	100%
	0%	105.000	105.092	105.075	105.000	104.891	104.760
	20%		84.000	84.038	84.000	83.918	83.808
	40%			63.000	63.000	62.945	62.856
	60%				42.000	41.973	41.904
	80%					21.000	20.952
	100%						0.000

Monopoly Block

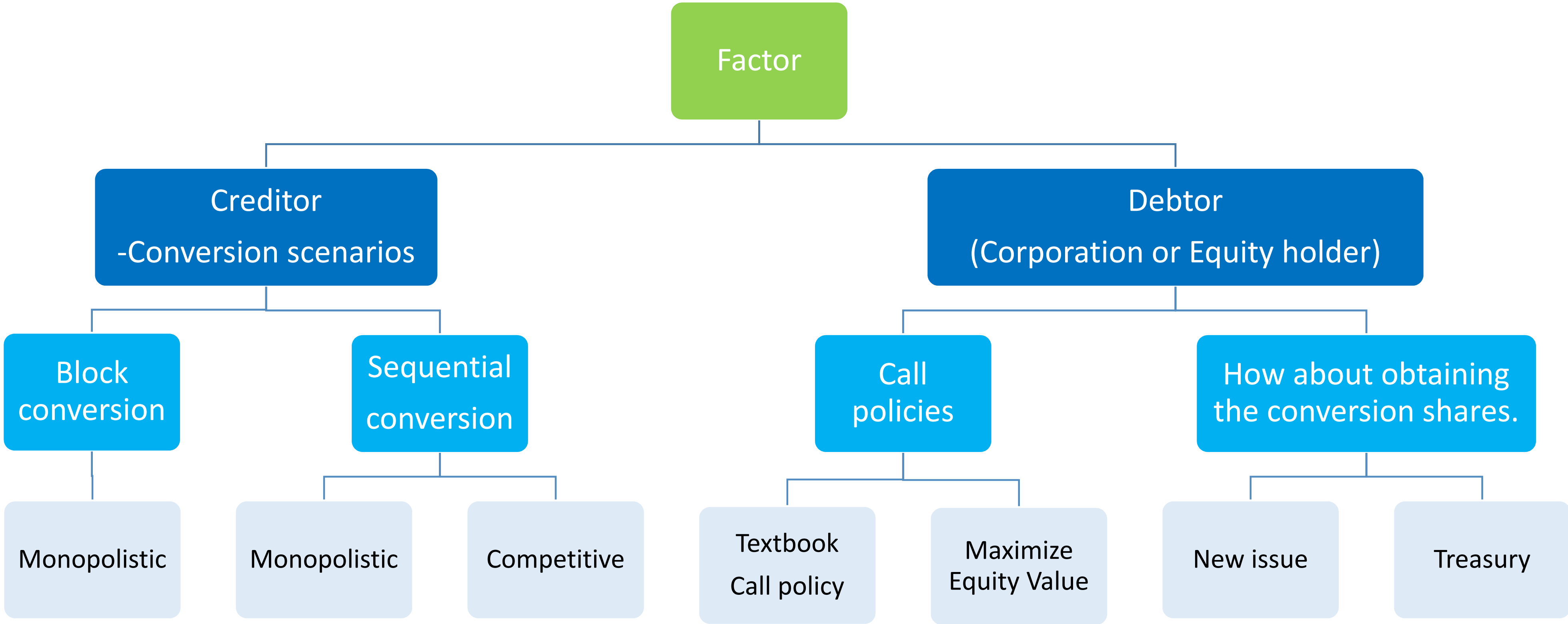
Monopoly Sequential conversion

Competitive Sequential conversion

Basic assumptions

- Asset = convertible bond + straight bond + equity
- The convertible bonds have the same maturity and issue day as Straight bond.
- When the bankrupt, the Straight bond ordinary is higher than Convertible bond.
- Treasury share equal Convertible bond conversion each period.
- If payout more than coupon then the extra part is the dividend.
- If payout less than coupon that is not enough part of the coupon is paid by issuing new shares

Binomial tree model: Factor



Binomial tree model：Building asset tree

Base Brodie and Kaya(2007) model assume $V_0 = x_1, \sigma_v = x_2$ given the initial value of x_1, x_2.
According to Cox, Ross and Rubinstein (1979), the tree structure simulates the company's assets .

The Using the following two restricted and matlab function nonlinear programming to calculate the model corresponding to the initial stock price $S_{\mathrm{mod}el}$:

$$S_{\mathrm{mod}el} = S_0$$
$$S_{\mathrm{mod}el}\sigma_S = \frac{\Delta S}{\Delta V} V_0 \sigma_V$$

At this time, x_1 is the firm's initial assets V_0. x_2 is the company's asset volatilityσ_v.

Binomial tree model: Competitive

Reference **Constantinides (1984)** for the definition of perfect competition

- The number of holders: make the holders can not collusion with each other
- Individual holders hold a small share ：Whether the holder's performance or not does not affect the conversion of the share price

Competitive conversion formula

$$\gamma S_{before} + \text{accrued interest in hold period} \geq CB_{continue}$$

- The bondholder has a conversion value greater than the value of non-conversion on competitive, the bondholder will convert until all bonds are converted or equal.
- When the equal sign is established, bond holders are not preferred for conversion (indifference point).

Binomial tree model: Call policies

- Textbook Call policy (Minimize CB Value)
  - Named by **Longstaff and Tuckman(1994)**
  - Convertible bond will Call when Call Price less than the convertible bond continuous value.
  - Continuous value of convertible bond that present value in the current period if the convertible corporate bonds are not converted or redeemed .

- Maximize Equity Value
  - Convertible bond will Call when the redemption corporation equity is better than not redemption.

Binomial tree model: How about obtaining the conversion shares

- **New Issue:** Shares outstanding is increase after the conversion

$$S_{before-convert} = \frac{V - D_{other}}{N_0}, \quad S_{After-convert} = \frac{V - D_{other}^{new}}{N_0 + N_0^{new}}$$

- **Treasury:** Shares outstanding is constant

  Perfect market assumptions: Conditions under which the law of one price holds. The assumptions include frictionless markets, rational investors, and equal access to market prices and information.

$$S_{After-convert} = \frac{V - D_{other} - S_{before-convert} \times N_0'}{N_0}$$

$S_{After-convert}$:Stock price after the conversion , $S_{before-convert}$: Stock price before the conversion
V:Asset Value, D_{other}: debt value without convertible bonds, N_0: The original outstanding shares , N_0':shares of conversion, N_0^{new}: The new shares resulting from the conversion of convertible bonds

Binomial tree model: Other symbol

	Value	Face value or Price	Amount	Coupon
Straight bond	SB_i i=0,1,…,n	F_B	N_B	C_B
Convertible bond	CB_i i=0,1,…,n	F_C	N_C	C_C
Equity	$Equity_i$ i=0,1,…,n	S_i i=0,1,…,n	N_0	

γ ： conversion ratio

τ ： tax rate

ρ ： bankruptcy cost

n ： number of periods

Δt ： time interval (T/n)

α_i ： optimum conversion ratio of monopoly at i period

β_i ： optimum conversion ratio of competitive at i period

CP_{clean} ： call price (excluding accrued interest)

δ_i ： payout at i period

Backward Induction：Equity(Treasury)

- Maturity Date
  - Call

    $Equity = Asset + payout -$ Value of straight bond $-$ Call value $-$ Accrued interest $-$ Treasury costs

$$Equity = V_n + \delta_n - N_B F_B(1 + (1-\tau)C_B \Delta t) - (1 - y\%)N_C CP_{clean} - (1 - \mathrm{x}\%)N_C F_C (1-\tau)C_c \Delta t - (y\% - x\%)N_c \gamma S$$

The zero equity mean company default, so we only consider equity is a positive number.

$$Equity_{n,call}(x\%, y\%) = max(Equity, 0)$$

Equity and share price derived as follows:

The share price immediately before the conversion should equal the converted share price . By solving a linear equations that stock value:

➢ $\frac{Equity_{\mathrm{n},call}(x\%,y\%)}{N_0} = S_n$

$\Rightarrow V_n + \delta_n - N_B F_B(1 + (1-\tau)C_B \Delta t) - (1 - y\%)N_C CP_{clean} - (1 - \mathrm{x}\%)N_C F_C (1-\tau)C_c \Delta t - (y\% - x\%)N_c \gamma S_n = N_0 S_n$

Backward Induction：Equity(Treasury)

- Maturity Date
  - Call

$Equity = Asset + payout -$ Value of straight bond − Call value − Accrued interest − Treasury costs

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➢ $\frac{Equity_{n,call}(x\%,y\%)}{N_0} = S_n$

$$S_{After-convert} = \frac{V - D_{other} - S_{before-convert} \times N_0'}{N_0}$$

$\Rightarrow V_n + \delta_n - N_B F_B(1 + (1-\tau)C_B \Delta t) - (1 - y\%)N_C CP_{clean} - (1 - x\%)N_C F_C(1-\tau)C_c \Delta t - (y\% - x\%)N_c \gamma S_n = N_0 S_n$

Backward Induction：Equity(Treasury)

- Maturity Date
  - Call

$\Rightarrow V_n + \delta_n - N_B F_B(1 + (1-\tau)C_B \Delta t) - (1 - y\%)N_C CP_{clean} - (1 - \text{x}\%)N_C F_C(1-\tau)C_c \Delta t - (y\% - x\%)N_c \gamma S_n = N_0 S_n$

$\Rightarrow V_n + \delta_n - N_B F_B(1 + (1-\tau)C_B \Delta t) - (1 - y\%)N_C CP_{clean} - (1 - \text{x}\%)N_C F_C(1-\tau)C_c \Delta t = S_n \times (\, N_0 + (y\% - x\%)N_c \gamma)$

$\Rightarrow S_n = \frac{V_n + \delta_n - N_B F_B(1+(1-\tau)C_B \Delta t) - (1-y\%)N_C CP_{clean} - (1-\text{x}\%)N_C F_C(1-\tau)C_c \Delta t}{N_0 + (y\% - x\%)N_c \gamma}$

We can rewrite the $Equity_{\text{n},call}(x\%, y\%)$ Because the number of shares outstanding unchanged

$\Rightarrow Equity_{n,call}(x\%, y\%) = max(\frac{V_n + \delta_n - N_B F_B(1+(1-\tau)C_B \Delta t) - (1-y\%)N_C CP_{clean} - (1-\text{x}\%)N_C F_C(1-\tau)C_c \Delta t}{N_0 + (y\% - x\%)N_c \gamma} \times N_0, 0)$

Backward Induction：Equity(Treasury)

Maximize Equity

- Maturity Date
  - Uncall

$Equity$
$= Asset + payout -$ Value of straight bond $-$ Value of convertible bond(unconverted)
$-$ Accrued interest $-$ Treasury costs

$$Equity_{\text{n},uncall}(x\%, y\%)$$
$$= \max(V_n + \delta_n - N_B F_B(1 + (1-\tau)C_B \Delta t) - (1 - \text{y}\%)N_C F_C(1 + (1-\tau)C_c \Delta t)$$
$$-(y\% - x\%)N_c F_C(1-\tau)C_c \Delta t - (y\% - x\%)N_c \gamma S), 0)$$

The share price immediately before the conversion should equal the converted share price. By solving linear equations and consider number of shares outstanding unchanged that equity value:

$$Equity_{\text{n},uncall}(x\%, y\%) = \max\left(\frac{V_n + \delta_n - N_B F_B(1+(1-\tau)C_B \Delta t) - (1-\text{y}\%)N_C F_C - (1-x\%)N_c F_C(1-\tau)C_c \Delta t}{N_0 + (y\% - x\%)N_c \gamma} \times N_0, 0\right)$$

Backward Induction：Equity(Treasury)

- Maturity Date
  - Called

    $Equity = Asset + payout -$ Value of straight bond

  - $Equity_{\mathrm{n},called}(x\%, \mathrm{y}\%) = \max(V_n + \delta_n - N_B F_B(1 + (1-\tau)C_B \Delta t), 0)$

    note：x=y here

Backward Induction：Stock price(Treasury)

- Before Maturity

$$Stock_n^*(x\%, y\%) = \begin{cases} \dfrac{Equity_n^*(x\%, y\%)}{N_0}, & if\ Equity > 0 \\ 0, & Otherwise \end{cases}$$

$$Stock_i^*(x\%, y\%) \neq \begin{cases} \dfrac{Equity_i^*(x\%, y\%)}{N_0}, & if\ Equity > 0 \\ 0, & Otherwise \end{cases}$$

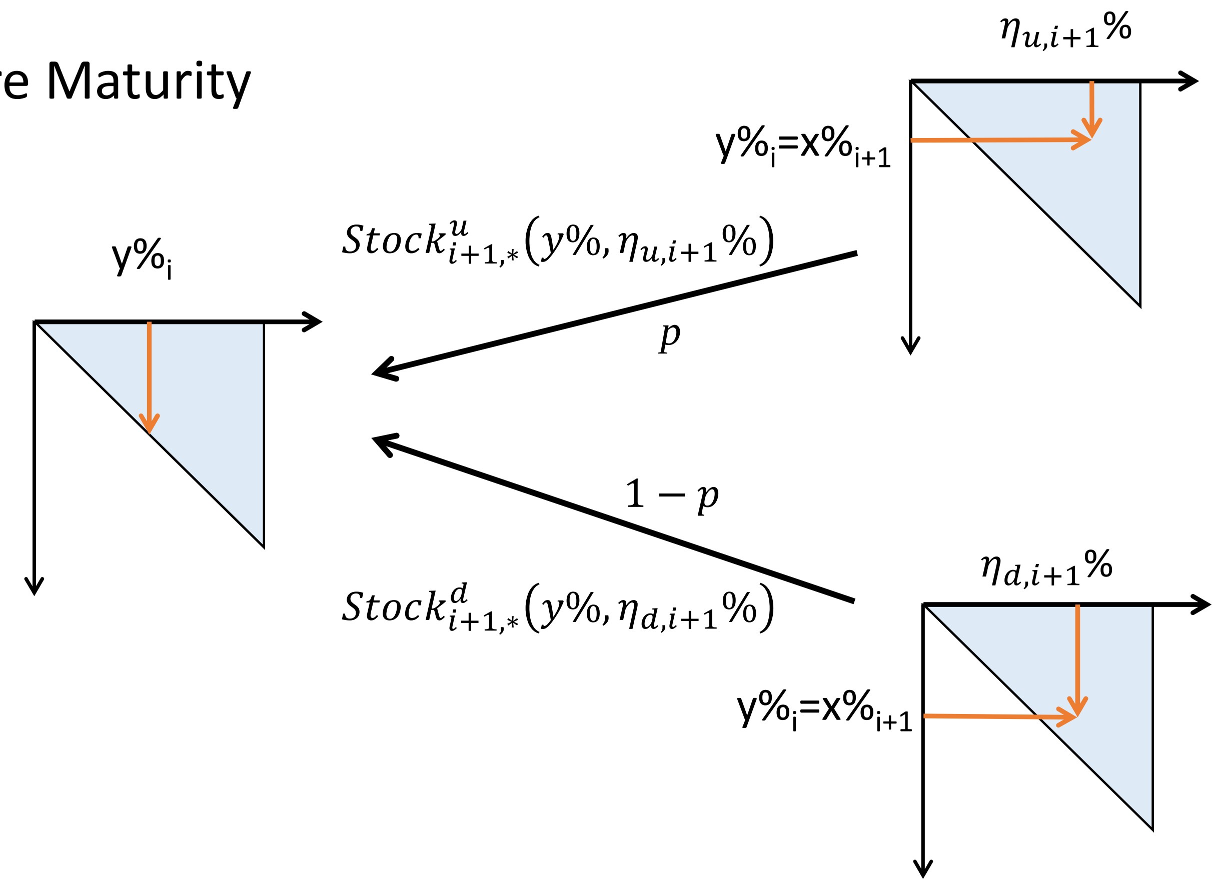
η is optimum conversion ratio. According to Block, Monopolistic and Competitive case.

* Is Call, Uncall, Called. If i period is Call or Called then i+1 period is Called, if i period is Uncall then i+1 period is call or Uncall.

$$PV\_Stock_{i,(*)} = \left(p \times Stock_{i+1,*}^{u}\left(y\%, \eta_{u,i+1}\%\right) + (1-p) \times Stock_{i+1,*}^{d}\left(y\%, \eta_{d,i+1}\%\right)\right) \times e^{-r_f \Delta t}$$

Backward Induction：Stock price(Treasury)

- Before Maturity



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$$PV\_Stock_{i,(*)} = \left(p \times Stock^{u}_{i+1,*}(y\%, \eta_{u,i+1}\%) + (1-p) \times Stock^{d}_{i+1,*}(y\%, \eta_{d,i+1}\%)\right) \times e^{-r_f \Delta t}$$

Backward Induction：Equity(Treasury)

- Before Maturity
  - Call

coupon of straight bond | Call value | Accrued interest | Treasury costs

$$Equity_{i,call}(x\%, y\%) = \max(N_0 \times (PV\_Stock) + \delta_i - N_B F_B (1-\tau) C_B \Delta t - (1-y\%) N_C \mathrm{CP}_{clean} - (1-\mathrm{x}\%) N_C F_C (1-\tau) C_c \Delta t - (y\% - x\%) N_C \gamma S_i, 0)$$

$$PV\_Stock = \left(p \times Stock^{u}_{i+1,called}(y\%, \eta_{u,i+1}\%) + (1-p) \times Stock^{d}_{i+1,called}(y\%, \eta_{d,i+1}\%)\right) \times e^{-r_f \Delta t}$$

Consider the stock price and rewrite:

$$Equity_{i,call}(x\%, y\%) = \max\left(\frac{N_0 \times (PV\_Stock) + \delta_i - N_B F_B (1-\tau) C_B \Delta t - (1-y\%) N_C \mathrm{CP}_{clean} - (1-\mathrm{x}\%) N_C F_C (1-\tau) C_c \Delta t}{N_0 + (y\% - x\%) N_C \gamma} \times N_0, 0\right)$$

  - Uncall

$$Equity_{i,uncall}(x\%, y\%) = \max\left(\frac{N_0 \times (PV\_Stock) + \delta_i - N_B F_B (1-\tau) C_B \Delta t - (1-\mathrm{x}\%) N_C F_C (1-\tau) C_C \Delta t}{N_0 + (y\% - x\%) N_C \gamma} \times N_0, 0\right)$$

  - Called

$$Equity_{i,called}(x\%, \mathrm{y}\%) = \max(N_0 \times (PV\_Stock) + \delta_i - N_B F_B (1-\tau) C_B \Delta t, 0)$$

note：x=y here

Backward Induction：Equity(Treasury)

- Issue day
  - Issue date doesn't need to consider coupon, accrued interest and payout δ.
  - **Call** $Equity_{0,call}(0\%, y\%) = \max(N_0 \times (PV\_Stock) - (1 - y\%)N_C CP_{clean} - (y\% - 0\%)N_c \gamma S_0, 0)$
  - **Uncall**

$$Equity_{0,uncall}(0\%, y\%)$$
$$= max\left(N_0 \times \left(p \times Stock^u_{i+1,*}(y\%, \eta_{u,i+1}) + q \times Stock^d_{i+1,*}(y\%, \eta_{d,i+1})\right) \times e^{-r_f \Delta t} - (y\% - 0\%)N_c \gamma S_0, 0\right)$$

Backward Induction：Equity(Treasury)

- Issue day
  - Issue date doesn't need to consider coupon, accrued interest and payout δ.
  - **Call**

$$Equity_{0,call}(0\%, y\%) = \begin{cases} \left(\frac{N_0 \times (PV\_Stock) - (1-y\%)N_C CP_{clean}}{N_0 + (y\%)N_c\gamma}\right) \times N_0, & if\ Equity_{0,call} > 0 \\ 0, & Otherwise \end{cases}$$

  - **Uncall**

$$Equity_{0,\mathrm{uncall}}(0\%, y\%) = \begin{cases} \frac{\left(p \times Stock^{u}_{i+1,*}(y\%, \eta_{u,i+1}) + (1-p) \times Stock^{d}_{i+1,*}(y\%, \eta_{d,i+1})\right) \times e^{-r_f \Delta t}}{N_0 + (y\%)N_c\gamma} \times N_0, & if\ Equity_{0,\mathrm{uncall}} > 0 \\ 0, & Otherwise \end{cases}$$

Backward Induction：Equity (New Issue)

- Before Maturity
  - Uncall
    - Stock price of Treasury

Shares outstanding × Discount of expected stock price

Coupon of straight bond

Accrued interest

$$S_i = \frac{N_0 \times (PV\_Stock) + \delta_i - N_B F_B (1-\tau) C_B \Delta t - (1-x\%) N_C F_C (1-\tau) C_c \Delta t}{N_0 + (y\% - x\%) N_c \gamma}$$

    - Stock price of New issue

$$S_i = \frac{(N_0 + y\% N_c \gamma) \times (PV\_Stock) + \delta_i - N_B F_B (1-\tau) C_B \Delta t - (1-x\%) N_C F_C (1-\tau) C_c \Delta t}{N_0 + y\% N_c \gamma}$$

  - The formula is similar like treasury. Differences are stock price and treasury costs.

Backward Induction：Equity (New Issue)

$$S_{before-convert} = \frac{V - D_{other}}{N_0}, S_{After-convert} = \frac{V - D_{other}^{new}}{N_0 + N_0^{new}}$$

- Before Maturity
  - Uncall
    - Stock price of New issue
    - $Equity_{i,uncall}(x\%, y\%) = \max(N_0 \times (PV\_Stock) + \delta_i - N_B F_B (1-\tau) C_B \Delta t - (1 - \text{x}\%) N_C F_C (1-\tau) C_c \Delta t, 0)$

$$\Rightarrow S_{before-convert} = \frac{N_0 \times (PV\_Stock) + \delta_i - N_B F_B (1-\tau) C_B \Delta t - (1-\text{x}\%) N_C F_C (1-\tau) C_c \Delta t}{N_0}$$

$$\Rightarrow S_{After-convert} = \frac{(N_0 + \text{y}\% N_c \gamma) \times (PV\_Stock) + \delta_i - N_B F_B (1-\tau) C_B \Delta t - (1-\text{x}\%) N_C F_C (1-\tau) C_c \Delta t}{N_0 + \text{y}\% N_c \gamma}$$

$$S_i = \frac{(N_0 + \text{y}\% N_c \gamma) \times (PV\_Stock) + \delta_i - N_B F_B (1-\tau) C_B \Delta t - (1-\text{x}\%) N_C F_C (1-\tau) C_c \Delta t}{N_0 + y\% N_c \gamma}$$

Backward Induction：Convertible Bond

- **Maturity Date**
  - $CB_n(x\%, y\%) = \begin{cases} \text{Value of remaining convertible bond} + \text{Accrued interest} + \text{conversion value}, Equity > 0 \\ (Asset + payout) - bankruptcy\ cost - \text{Value of straight bond}\ , Otherwise \end{cases}$
  - **Call**

$$CB_n(x\%, y\%) =$$

$$\begin{cases} (1 - y\%)N_C \mathrm{CP}_{clean} + (1 - \mathrm{x}\%)N_C F_C C_c \Delta t + (y\% - x\%) \times \gamma \times N_C \times Stock_{n,call}(x\%, y\%) & ,if \quad Equity_{n,call}(x\%, y\%) > 0 \\ \max((1 - \rho) \times (V_n + \delta_n) - N_B \times F_B \times (1 + C_B \Delta t) \quad , \quad 0) & , \quad Otherwise \end{cases}$$

  - **Uncall**

$$CB_n(x\%, y\%) =$$

$$\begin{cases} (1 - \mathrm{y}\%)N_C F_C + (1 - x\%)N_c F_C C_c \Delta t + (y\% - x\%) \times \gamma \times N_C \times Stock_{n,uncall}(x\%, y\%) & ,if \quad Equity_{n,uncall}(x\%, y\%) > 0 \\ \max((1 - \rho) \times (V_n + \delta_n) - N_B \times F_B \times (1 + C_B \Delta t) \quad , \quad 0) & , \quad Otherwise \end{cases}$$

Backward Induction： Convertible Bond

- **Before Maturity**
  - **Call**

$$CB_i(x\%, y\%) = \begin{cases} (1-y\%)N_C \mathrm{CP}_{clean} + (1-\mathrm{x}\%)N_C F_C C_c \Delta t + (y\% - x\%) \times \gamma \times N_C \times Stock_{i,call}(x\%, y\%) & ,if \quad Equity_{i,call}(x\%, y\%) > 0 \\ \max((1-\rho) \times (V_i + \delta_i) - PV\_SBCF \quad , \quad 0) & , \quad Otherwise \end{cases}$$

PV_SBCF is the present value of cash flows in the future of straight bond.

  - **Uncall**

$$CB_i(x\%, y\%) = \begin{cases} CB_{i,continuation}^{total} + (y\% - x\%)N_c F_C C_c \Delta t + (y\% - x\%) \times \gamma \times N_C \times Stock_{i,uncall}(x\%, y\%) & ,if \quad Equity_{i,uncall}(x\%, y\%) > 0 \\ \max((1-\rho) \times (V_i + \delta_i) - PV\_SBCF \quad , \quad 0) & , \quad Otherwise \end{cases}$$

$$CB_{i,continuation}^{total} = \left(p \times CB_{u,i+1}(y\%, \eta_{u,i+1}\%) + (1-p) \times CB_{d,i+1}(y\%, \eta_{d,i+1}\%)\right) \times e^{-r_f \Delta t} + (1-y\%) \times N_C \times F_C \times C_C \Delta t$$

Textbook policy: $(1-y\%)N_C(\mathrm{CP}_{i,clean} + F_C C_c \Delta t) \leq CB_{i,continuation}^{total}$

Backward Induction： Convertible Bond

- **Issue day**
  - The difference between "Issue date" and "Before Maturity" are not need to pay coupon and accrued interest.
  - **Call**

$$CB_0(0\%, y\%) = \begin{cases} y\% \times N_C \times CP_{clean} + y\% \times \gamma \times N_C \times Stock_{0,call}(0\%, y\%) & ,if \quad Equity_{0,call}(0\%, y\%) > 0 \\ \max((1-\rho) \times V_0 - PV\_SBCF \quad , \quad 0) & , \quad Otherwise \end{cases}$$

  - **Uncall**

$$CB_0(0\%, y\%) = \begin{cases} CB_{0,continuation}^{total} + y\% \times \gamma \times N_C \times Stock_{0,uncall}(0\%, y\%) & ,if \quad Equity_{0,uncall}(0\%, y\%) > 0 \\ \max((1-\rho) \times V_0 - PV\_SBCF \quad , \quad 0) & , \quad Otherwise \end{cases}$$

PV_SBCF is the present value of cash flows in the future of straight bond.

$$CB_{0,continuation}^{total} = \left(p \times CB_{u,i+1}\left(y\%, \eta_{u,i+1}\%\right) + (1-p) \times CB_{d,i+1}\left(y\%, \eta_{d,i+1}\%\right)\right) \times e^{-r_f \Delta t}$$

Backward Induction： Straight Bond

- **Maturity day**

$$SB_n(x\%, y\%) = \begin{cases} N_B \times F_B \times (1 + C_B \Delta t) \ , & if \quad Equity_{n,*}(x\%, y\%) > 0 \\ \min\big((1-\rho) \times (V_n + \delta_n), N_B \times F_B \times (1 + C_B \Delta t)\big) \ , & Otherwise \end{cases}$$

- **Before Maturity**

$$SB_i(x\%, y\%) =$$

$$\begin{cases} \big(p \times SB_{i+1}^{u}(y\% \ ,\bullet) + (1-p) \times SB_{i+1}^{d}(y\% \ ,\bullet)\big) \times e^{-r_f \Delta t} + N_B \times F_B \times C_B \Delta t \ , & if \quad Equity_{i,*}(x\%, y\%) > 0 \\ \min\big((1-\rho) \times (V_i + \delta_i), PV\_SBCF\big) \ , & Otherwise \end{cases}$$

- **Issue day**

$$SB_0(0\%, y\%) = \begin{cases} \big(p \times SB_{i+1}^{u}(y\% \ ,\bullet) + (1-p) \times SB_{i+1}^{d}(y\% \ ,\bullet)\big) \times e^{-r_f \Delta t} \ , & if \quad Equity_{0,*}(0\%, y\%) > 0 \\ \min\big((1-\rho) \times (V_0 + \delta_0), PV\_SBCF\big) \ , & Otherwise \end{cases}$$

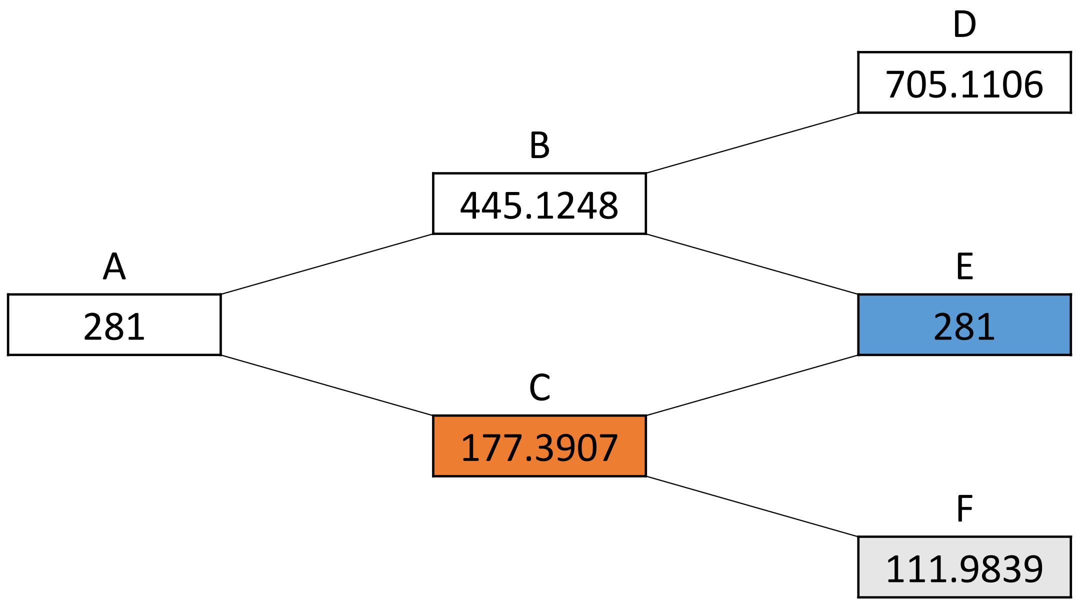
PV_SBCF is the present value of cash flows in the future of straight bond.
(•) is conversion ratio. According to the current situation can be y, α, β.
$*$ is $call, uncall, called$

Numerical method example

The parameters of this example					
Initial asset value	$V_{\mathbf{0}}$	281	Conversion ratio	γ	1.5
Asset volatility	σ_V	0.46	Risk-free interest rate	$r_{\boldsymbol{f}}$	0.06
Amount of Straight bond	N_B	1	Time to maturity	T	2
Amount of Convertible bond	N_C	1	Period	n	2
Shares outstanding	N_0	1	Time interval	$\Delta_{\boldsymbol{t}}$	1
Face value of Straight bond	F_B	100	Bankruptcy cost	ρ	0.4
Face value of Convertible bond	F_C	100	call price (excluding accrued interest)	CP_{clean}	115
Coupon of Straight bond	C_B	0.08	tax rate	τ	0.2
Coupon of Convertible bond	C_C	0.05	Payout rate	δ	0.03

Numerical method example

We can calculate up probability: $p = 0.4189493$ and down probability: $1 - p = 0.5810507$ by previous slide parameters.



Numerical method example：E Monopolistic, Treasury

Equity(uncall)	0%	20%	40%	60%	80%	100%
0%	79.1577	76.2752	74.4736	73.2409	72.3444	71.6631
20%		99.9577	92.2752	87.4736	84.1883	81.7990
40%			120.7577	108.2752	100.4736	95.1356
60%				141.5577	124.2752	113.4736
80%					162.3577	140.2752
100%						183.1577

Equity(call)	0%	20%	40%	60%	80%	100%
0%	64.1577	67.0444	68.8486	70.0830	70.9808	71.6631
20%		87.9577	85.3521	83.7236	82.6093	81.7990
40%			111.7577	103.6598	98.5986	95.1356
60%				135.5577	121.9675	113.4736
80%					159.3577	140.2752
100%						183.1577

Equity(called)	0%	20%	40%	60%	80%	100%
0%	183.1577					
20%		183.1577				
40%			183.1577			
60%				183.1577		
80%					183.1577	
100%						183.1577

$$Equity_{n,uncall}(x\%, y\%) = \max\left(\frac{V_n+\delta_n-N_BF_B(1+(1-\tau)C_B\Delta t)-(1-y\%)N_CF_C-(1-x\%)N_cF_C(1-\tau)C_c\Delta t}{N_0+(y\%-x\%)N_c\gamma} \times N_0, 0\right)$$

$$Equity_{n,call}(x\%, y\%) = \max\left(\frac{V_n+\delta_n-N_BF_B(1+(1-\tau)C_B\Delta t)-(1-y\%)N_CCP_{clean}-(1-x\%)N_CF_C(1-\tau)C_c\Delta t}{N_0+(y\%-x\%)N_c\gamma} \times N_0, 0\right)$$

$$Equity_{n,called}(x\%, y\%) = \max(V_n + \delta_n - N_BF_B(1 + (1-\tau)C_B\Delta t), 0)$$

$$Equity_{E,called}(20\%, 20\%) = \max(281 \times e^{(0.03\times1)} - 100 \times 1 \times (1 + (1-0.2) \times 0.08 \times 1), 0) = 183.1577$$

Numerical method example：E Monopolistic, Treasury

Equity(uncall)	60%
0%	73.2409
20%	87.4736
40%	108.2752
60%	141.5577
80%	
100%	
Equity(call)	**60%**
0%	70.0830
20%	83.7236
40%	103.6598
60%	135.5577
80%	
100%	

$$Equity_{n,uncall}(x\%, y\%) = \max\left(\frac{V_n+\delta_n-N_BF_B(1+(1-\tau)C_B\Delta t)-(1-y\%)N_CF_C-(1-x\%)N_cF_C(1-\tau)C_c\Delta t}{N_0+(y\%-x\%)N_c\gamma} \times N_0, 0\right)$$

$$Equity_{E,uncall}(20\%, 60\%) = \max\left(\frac{281\times e^{(0.03\times1)}-100\times1\times(1+(1-0.2)\times0.08\times1)-(1-60\%)\times1\times100-(1-20\%)\times1\times100\times(1-0.2)\times0.05\times1}{1+(60\%-20\%)\times1\times1.5}\times 1, 0\right)$$

$=87.4736$

$$Equity_{n,call}(x\%, y\%) = \max\left(\frac{V_n+\delta_n-N_BF_B(1+(1-\tau)C_B\Delta t)-(1-y\%)N_CCP_{clean}-(1-x\%)N_CF_C(1-\tau)C_c\Delta t}{N_0+(y\%-x\%)N_c\gamma} \times N_0, 0\right)$$

$$Equity_{E,call}(20\%, 60\%) = \max\left(\frac{281\times e^{(0.03\times1)}-100\times1\times(1+(1-0.2)\times0.08\times1)-(1-60\%)\times1\times115-(1-20\%)\times1\times100\times(1-0.2)\times0.05\times1}{1+(60\%-20\%)\times1\times1.5}\times, 0\right)$$

$=83.7236$

Numerical method example：E Monopolistic, Treasury

Equity(uncall)	0%	20%	40%	60%	80%	100%
0%	79.1577	76.2752	74.4736	73.2409	72.3444	71.6631
20%		99.9577	92.2752	87.4736	84.1883	81.7990
40%			120.7577	108.2752	100.4736	95.1356
60%				141.5577	124.2752	113.4736
80%					162.3577	140.2752
100%						183.1577
Equity(call)	**0%**	**20%**	**40%**	**60%**	**80%**	**100%**
0%	64.1577	67.0444	68.8486	70.0830	70.9808	71.6631
20%		87.9577	85.3521	83.7236	82.6093	81.7990
40%			111.7577	103.6598	98.5986	95.1356
60%				135.5577	121.9675	113.4736
80%					159.3577	140.2752
100%						183.1577

Maximize Equity Call policy : Convertible bond will Call when the redemption corporation equity is better than not redemption.

Textbook Call policy :$(1-y\%)N_C\left(\mathrm{CP}_{i,clean}+F_C C_c \Delta t\right) \le CB_{i,continuation}^{total}$

$$CB_{n,continuation}^{total} = F_c \times (1+C_c) = 100 \times (1+0.05) = 105$$

$$(1-y\%)N_C\left(\mathrm{CP}_{n,clean}+F_c C_C \Delta t\right) =$$
$$(1-60\%) \times 1 \times (115 + 100 \times 0.05 \times 1) =$$
$$40\% \times 120 = 48$$

Numerical method example：E Monopolistic, Treasury

CB Value	0%	20%	40%	60%	80%	100%
0%	105	107.8826	109.6841	110.9168	111.8133	**112.4946**
20%		84	91.6826	96.4841	99.7694	**102.1588**
40%			63	75.4826	83.2841	**88.6221**
60%				42	59.2826	**70.0841**
80%					21	**43.0826**
100%						**0**

The monopolistic and competitive case have same optimum conversion ratio.

$$CB_{n,uncall}(x\%, y\%) =$$

$$\begin{cases} (1-y\%)N_C F_C + (1-x\%)N_c F_C C_c \Delta t + (y\% - x\%) \times \gamma \times N_C \times Stock_{n,uncall}(x\%, y\%) & ,if \quad Equity_{n,uncall}(x\%, y\%) > 0 \\ \max((1-\rho) \times (V_n + \delta_n) - N_B \times F_B \times (1 + C_B \Delta t) \quad , \quad 0) & , \quad Otherwise \end{cases}$$

$$CB_E(20\%, 60\%)$$
$$= (1-60\%) \times 1 \times 100 + (1-20\%) \times 1 \times 100 \times 0.05 \times 1 + (60\% - 20\%) \times 1.5 \times 1 \times 87.4736$$
$$= 96.4841$$

Numerical method example：E Competitive, Treasury

The converted value	0%	20%	40%	60%	80%	100%
0%	123.74	119.41	116.71	114.86	113.52	112.49
20%		154.94	143.41	136.21	131.28	127.70
40%			186.14	167.41	155.71	147.70
60%				217.34	191.41	175.21
80%					248.54	215.41
100%						279.74
Unconverted value	105	105	105	105	105	105
	0%	20%	40%	60%	80%	100%
0%	1	1	1	1	1	1
20%		1	1	1	1	1
40%			1	1	1	1
60%				1	1	1
80%					1	1
100%						1

$$CB_{n,continuation}^{total} = F_c \times (1 + C_c) = 105$$

$$\gamma \times Stock_E(x\%, y\%) + F_c C_C \Delta t \geq CB_{n,continuation}^{total}$$

Numerical method example：E Monopolistic, Treasury

SB Value (uncall)	0%	20%	40%	60%	80%	100%
0%	108	108	108	108	108	108
20%		108	108	108	108	108
40%			108	108	108	108
60%				108	108	108
80%					108	108
100%						108
SB Value (call)	0%	20%	40%	60%	80%	100%
0%	108	108	108	108	108	108
20%		108	108	108	108	108
40%			108	108	108	108
60%				108	108	108
80%					108	108
100%						108
SB Value (called)	0%	20%	40%	60%	80%	100%
0%	108					
20%		108				
40%			108			
60%				108		
80%					108	
100%						108

$$SB_E(x\%, y\%) = N_B \times F_B \times (1 + C_B \Delta t)$$
$$\Rightarrow 1 \times 100 \times (1 + 0.08 \times 1) = 108$$

Numerical method example：F Monopolistic, Treasury

SB Value (uncall)	0%	20%	40%	60%	80%	100%
0%	69.2366	69.2366	69.2366	69.2366	69.2366	108
20%		69.2366	69.2366	69.2366	69.2366	108
40%			69.2366	69.2366	69.2366	108
60%				69.2366	69.2366	108
80%					69.2366	108
100%						108
SB Value (call)	0%	20%	40%	60%	80%	100%
0%	69.2366	69.2366	69.2366	69.2366	69.2366	108
20%		69.2366	69.2366	69.2366	69.2366	108
40%			69.2366	69.2366	69.2366	108
60%				69.2366	69.2366	108
80%					69.2366	108
100%						108
SB Value (called)	0%	20%	40%	60%	80%	100%
0%	108					
20%		108				
40%			108			
60%				108		
80%					108	
100%						108

$$SB_E(x\%, y\%) = N_B \times F_B \times (1 + C_B \Delta t)$$
$$\Rightarrow 1 \times 100 \times (1 + 0.08 \times 1) = 108$$

$$SB_F(x\%, y\%) =$$
$$\min\big((1-\rho) \times (V_n + \delta_n), N_B \times F_B \times (1 + C_B \Delta t)\big)$$
$$\Rightarrow min\left((1-0.4) \times 111.9839 \times e^{(0.03\times1)}, 108\right)$$

Numerical method example：F Monopolistic, Treasury

Equity(uncall)	0%	20%	40%	60%	80%	100%
0%	0	0	0	0	0	1.998
20%		0	0	0	0	2.634
40%			0	0	0	3.471
60%				0	0	4.621
80%					0	6.303
100%						8.994
Equity(call)	0%	20%	40%	60%	80%	100%
0%	0	0	0	0	0	1.998
20%		0	0	0	0	2.634
40%			0	0	0	3.471
60%				0	0	4.621
80%					0	6.303
100%						8.994
Equity(called)	0%	20%	40%	60%	80%	100%
0%	8.99					
20%		8.99				
40%			8.99			
60%				8.99		
80%					8.99	
100%						8.99

Numerical method example：F Monopolistic and Competitive, Treasury

CB Value	0%	20%	40%	60%	80%	100%
0%	0	0	0	0	0	**7.997**
20%		0	0	0	0	**7.161**
40%			0	0	0	**6.124**
60%				0	0	**4.773**
80%					0	**2.891**
100%						**0**

The converted value	0%	20%	40%	60%	80%	100%
0%	0	0	0	0	0	7.9966
20%		0	0	0	0	8.9506
40%			0	0	0	10.2060
60%				0	0	11.9321
80%					0	14.4549
100%						18.4914
Unconverted value	0	0	0	0	0	105
	0%	20%	40%	60%	80%	100%
0%	0	0	0	0	0	0
20%		0	0	0	0	0
40%			0	0	0	0
60%				0	0	0
80%					0	0
100%						0

Numerical method example：C
Monopolistic and Competitive, Treasury and New Issue

Monopolistic (Treasury)						
Equity (uncall)	0%	20%	40%	60%	80%	100%
0%	24.37	22.09	21.52	22.26	24.45	28.88
20%		29.52	27.11	26.94	28.74	33.18
40%			36.04	33.77	34.62	38.84
60%				44.70	43.23	46.62
80%					57.00	57.99
100%						76.19
Equity (call)	0%	20%	40%	60%	80%	100%
0%	0.00	0.00	1.99	13.78	22.36	28.88
20%		0.00	3.07	16.87	26.31	33.18
40%			4.79	21.38	31.74	38.84
60%				28.59	39.68	46.62
80%					52.39	57.99
100%						76.19
Equity (called)	0%	20%	40%	60%	80%	100%
0%	76.19					
20%		76.19				
40%			76.19			
60%				76.19		
80%					76.19	
100%						76.19

Competitive (Treasury)						
Equity (uncall)	0%	20%	40%	60%	80%	100%
0%	23.28	20.98	20.34	20.93	22.89	28.88
20%		28.08	25.64	25.36	26.92	33.18
40%			34.14	31.83	32.47	38.84
60%				42.17	40.58	46.62
80%					53.55	57.99
100%						76.19
Equity (call)	0%	20%	40%	60%	80%	100%
0%	0.00	0.00	1.99	13.78	22.36	28.88
20%		0.00	3.07	16.87	26.31	33.18
40%			4.79	21.38	31.74	38.84
60%				28.59	39.68	46.62
80%					52.39	57.99
100%						76.19
Equity (called)	0%	20%	40%	60%	80%	100%
0%	76.19					
20%		76.19				
40%			76.19			
60%				76.19		
80%					76.19	
100%						76.19

Monopolistic (New Issue)						
Stock price (uncall)	0%	20%	40%	60%	80%	100%
0%	**24.37**	25.82	26.85	27.64	28.30	**28.88**
20%		26.44	27.35	28.06	28.67	29.20
40%			27.85	28.48	29.03	29.52
60%				28.90	29.39	29.84
80%					29.76	30.16
100%						30.48
Stock price (call)	0%	20%	40%	60%	80%	100%
0%	**0.00**	**0.00**	**1.99**	**13.78**	**22.36**	**28.88**
20%		0.00	2.49	14.20	22.72	29.20
40%			2.99	14.63	23.09	29.52
60%				15.05	23.45	29.84
80%					23.81	30.16
100%						30.48
Stock price (called)	0%	20%	40%	60%	80%	100%
0%	**76.19**					
20%		58.61				
40%			47.62			
60%				40.10		
80%					34.63	
100%						30.48

Numerical method example：C Monopolistic, Treasury

Equity(uncall)	60%
0%	22.26
20%	26.94
40%	33.77
60%	44.70
80%	
100%	
Equity(call)	60%
0%	13.78
20%	16.87
40%	21.38
60%	28.59
80%	
100%	
Equity(called)	60%
0%	
20%	76.19
40%	
60%	
80%	
100%	

$$Equity_{i,called}(x\%, y\%) = \max(N_0 \times (PV\_Stock) + \delta_i - N_B F_B (1-\tau) C_B \Delta t, 0)$$

$$Equity_{c,called}(x\%, y\%)$$
$$= \max\left(1 \times (77.17652) + 177.3907 \times \left(e^{(0.03\times1)} - 1\right) - 1 \times 100 \times (1-0.2) \times 0.08 \times 1, 0\right)$$
$$= 76.19$$

$$PV\_Stock = \left(p \times Stock^{u}_{i+1,called}(y\%, \eta_{u,i+1}\%) + (1-p) \times Stock^{d}_{i+1,called}(y\%, \eta_{d,i+1}\%)\right) \times e^{-r_f \Delta t}$$

$$77.17652 = (0.4189 \times 183.1577 + 0.5811 \times 8.99)e^{-0.06\times1}$$

Numerical method example：E Monopolistic, Treasury

Equity(uncall)	0%	20%	40%	60%	80%	100%
0%	79.1577	76.2752	74.4736	73.2409	72.3444	71.6631
20%		99.9577	92.2752	87.4736	84.1883	81.7990
40%			120.7577	108.2752	100.4736	95.1356
60%				141.5577	124.2752	113.4736
80%					162.3577	140.2752
100%						183.1577
Equity(call)	**0%**	**20%**	**40%**	**60%**	**80%**	**100%**
0%	64.1577	67.0444	68.8486	70.0830	70.9808	71.6631
20%		87.9577	85.3521	83.7236	82.6093	81.7990
40%			111.7577	103.6598	98.5986	95.1356
60%				135.5577	121.9675	113.4736
80%					159.3577	140.2752
100%						183.1577
Equity(called)	**0%**	**20%**	**40%**	**60%**	**80%**	**100%**
0%	183.1577					
20%		183.1577				
40%			183.1577			
60%				183.1577		
80%					183.1577	
100%						183.1577

Numerical method example：C Monopolistic, Treasury

Equity(uncall)	60%
0%	22.26
20%	26.94
40%	33.77
60%	44.70
80%	
100%	
Equity(call)	60%
0%	13.78
20%	16.87
40%	21.38
60%	28.59
80%	
100%	
Equity(called)	60%
0%	
20%	76.19
40%	
60%	
80%	
100%	

$$Equity_{\text{c,uncall}}(x\%, y\%) = \max\left(\frac{N_0 \times (PV\_Stock) + \delta_i - N_B F_B (1-\tau) C_B \Delta t - (1-\text{x}\%) N_C F_C (1-\tau) C_c \Delta t}{N_0 + (y\% - x\%) N_C \gamma} \times N_0, 0\right)$$

$$\text{Equity}_{\text{c,uncall}}(20\%,60\%) =$$

$$\max\left(\frac{1\times(47.295)+177.3907\times\left(\text{e}^{(0.03\times1)}-1\right)-1\times100\times(1-0.2)\times0.08\times1-(1-0.2)\times1\times100\times(1-0.2)\times0.05\times1}{1+(60\%-20\%)\times1\times1.5}\times1,0\right)$$

$$=26.94$$

$$PV\_Stock_{c,uncall}$$
$$= \left(p \times Stock^{u}_{i+1,uncall}\left(y\%, \eta_{u,i+1}\%\right) + (1-p) \times Stock^{d}_{i+1,uncall}\left(y\%, \eta_{d,i+1}\%\right)\right) \times e^{-r_f \Delta t}$$

$$PV\_Stock_{c,uncall} = (0.4189 \times 113.4736 + 0.5811 \times 4.621) e^{-0.06\times1} = 47.295$$

Numerical method example：E Monopolistic, Treasury

Equity(uncall)	0%	20%	40%	60%	80%	100%
0%	79.1577	76.2752	74.4736	73.2409	72.3444	71.6631
20%		99.9577	92.2752	87.4736	84.1883	81.7990
40%			120.7577	108.2752	100.4736	95.1356
60%				141.5577	124.2752	113.4736
80%					162.3577	140.2752
100%						183.1577

Equity(call)	0%	20%	40%	60%	80%	100%
0%	64.1577	67.0444	68.8486	70.0830	70.9808	71.6631
20%		87.9577	85.3521	83.7236	82.6093	81.7990
40%			111.7577	103.6598	98.5986	95.1356
60%				135.5577	121.9675	113.4736
80%					159.3577	140.2752
100%						183.1577

Equity(called)	0%	20%	40%	60%	80%	100%
0%	183.1577					
20%		183.1577				
40%			183.1577			
60%				183.1577		
80%					183.1577	
100%						183.1577

CB Value	0%	20%	40%	60%	80%	100%
0%	105	107.8826	109.6841	110.9168	111.8133	**112.4946**
20%		84	91.6826	96.4841	99.7694	**102.1588**
40%			63	75.4826	83.2841	**88.6221**
60%				42	59.2826	**70.0841**
80%					21	**43.0826**
100%						**0**

Numerical method example：C Monopolistic, Treasury

Equity(uncall)	60%
0%	22.26
20%	26.94
40%	33.77
60%	44.70
80%	
100%	
Equity(call)	60%
0%	13.78
20%	16.87
40%	21.38
60%	28.59
80%	
100%	
Equity(called)	60%
0%	
20%	76.19
40%	
60%	
80%	
100%	

$$Equity_{c,call}(x\%, y\%)$$

$$= \max\left(\frac{N_0 \times (PV\_Stock) + \delta_i - N_B F_B (1-\tau) C_B \Delta t - (1-y\%) N_C \mathrm{CP}_{clean} - (1-x\%) N_C F_C (1-\tau) C_c \Delta t}{N_0 + (y\% - x\%) N_c \gamma} \times N_0, 0\right)$$

$$\mathrm{Equity}_{\mathrm{c,call}}(20\%,60\%)=$$

$$\max\left(\frac{1\times(77.1765)+177.3907\times\left(e^{(0.03\times1)}-1\right)-1\times100\times(1-0.2)\times0.08\times1-(1-0.6)\times1\times115-(1-0.2)\times1\times100\times(1-0.2)\times0.05\times1}{1+(60\%-20\%)\times1\times1.5}\times1,\right.$$

$$=16.87$$

$$PV\_Stock_{c,call}$$

$$= \left(p \times Stock^{u}_{i+1,called}\left(y\%, \eta_{u,i+1}\%\right) + (1-p) \times Stock^{d}_{i+1,called}\left(y\%, \eta_{d,i+1}\%\right)\right) \times e^{-r_f \Delta t}$$

$$PV\_Stock_{c,call} = (0.4189 \times 183.1577 + 0.5811 \times 8.99)e^{-0.06\times1} = 77.17652$$

Numerical method example：C Monopolistic, New Issue

Stock price (uncall)	0%	20%	40%	60%	80%	100%
0%	**24.37**	25.82	26.85	27.64	28.30	**28.88**
20%		26.44	27.35	28.06	28.67	29.20
40%			27.85	28.48	29.03	29.52
60%				28.90	29.39	29.84
80%					29.76	30.16
100%						30.48

Stock price (call)	0%	20%	40%	60%	80%	100%
0%	**0.00**	**0.00**	**1.99**	**13.78**	**22.36**	**28.88**
20%		0.00	2.49	14.20	22.72	29.20
40%			2.99	14.63	23.09	29.52
60%				15.05	23.45	29.84
80%					23.81	30.16
100%						30.48

Stock price (called)	0%	20%	40%	60%	80%	100%
0%	**76.19**					
20%		58.61				
40%			47.62			
60%				40.10		
80%					34.63	
100%						30.48

$$S_{i,\text{uncall}}(x\%, y\%) = \frac{(N_0+y\%N_c\gamma)\times(PV\_Stock)+\delta_i-N_BF_B(1-\tau)C_B\Delta t-(1-x\%)N_CF_C(1-\tau)C_c\Delta t}{N_0+y\%N_c\gamma}$$

$$S_{i,\text{call}}(x\%, y\%) = \frac{(N_0+y\%N_c\gamma)\times(PV\_Stock)+\delta_i-N_BF_B(1-\tau)C_B\Delta t-(1-y\%)N_C\text{CP}_{clean}-(1-x\%)N_CF_C(1-\tau)C_c\Delta t}{N_0+y\%N_c\gamma}$$

$$S_{i,\text{called}}(x\%, y\%) = \frac{(N_0+y\%N_c\gamma)\times(PV\_Stock)+\delta_i-N_BF_B(1-\tau)C_B\Delta t-(1-x\%)N_CF_C(1-\tau)C_c\Delta t}{N_0+y\%N_c\gamma}$$

$$Stock_n^*(x\%, y\%) = \begin{cases} \dfrac{Equity_n^*(x\%, y\%)}{N_0 + y\%N_c\gamma}, & if\ Equity > 0 \\ 0, & Otherwise \end{cases}$$

Numerical method example：C Monopolistic, New Issue

Stock price (uncall)	0%	20%	40%	60%	80%	100%
0%	24.37	25.82	26.85	27.64	28.30	28.88
20%		26.44	27.35	28.06	28.67	29.20
40%			27.85	28.48	29.03	29.52
60%				28.90	29.39	29.84
80%					29.76	30.16
100%						30.48

$$S_{i,\text{uncall}}(x\%, y\%) = \frac{(N_0+\text{y}\%N_C\gamma)\times(PV\_Stock)+\delta_i-N_BF_B(1-\tau)C_B\Delta t-(1-\text{x}\%)N_CF_C(1-\tau)C_c\Delta t}{N_0+y\%N_C\gamma}$$

$$\text{S}_{\text{c,uncall}}(20\%,60\%) = \frac{(1+60\%\times1\times1.5)\times(\text{PV\_Stock})+177.3907\times\left(\text{e}^{(0.03\times1)}-1\right)-1\times100\times(1-0.2)\times0.08\times1-(1-0.2)\times1\times100\times(1-0.2)\times0.05\times1}{1+60\%\times1\times1.5}$$

$$= 28.06$$

$$PV\_Stock_{c,uncall} = (0.4189 \times 45.389 + 0.5811 \times 22.586)e^{-0.06\times1} = 30.27$$

Numerical method example：E Monopolistic, New Issue

Stock price (uncall)	0%	20%	40%	60%	80%	100%
0%	79.158	58.673	46.546	38.548	32.884	28.665
20%		76.891	57.672	46.039	38.267	32.720
40%			75.474	56.987	45.670	38.054
60%				74.504	56.489	45.389
80%					73.799	56.110
100%						73.263

Stock price (call)	0%	20%	40%	60%	80%	100%
0%	64.158	51.573	43.030	36.886	32.264	28.665
20%		67.660	53.345	44.065	37.550	32.720
40%			69.849	54.558	44.818	38.054
60%				71.346	55.440	45.389
80%					72.435	56.110
100%						73.263

Stock price (called)	0%	20%	40%	60%	80%	100%
0%	183.158					
20%		140.891				
40%			114.474			
60%				96.399		
80%					83.254	
100%						73.263

Numerical method example：C Monopolistic, Treasury

Monopolistic						
CB Value	0%	20%	40%	60%	80%	100%
0%	53.761	55.852	**56.231**	55.302	52.924	48.314
20%		48.225	**50.449**	50.427	48.442	43.812
40%			41.317	**43.395**	42.354	37.953
60%				32.264	**33.549**	29.971
80%					**19.580**	18.398
100%						**0.000**

Monopolistic						
Y%	0%	20%	40%	60%	80%	100%
$CB_{i,continuation}^{total}$	53.761	48.225	41.317	32.264	19.580	0.000
$\mathrm{CP}_{n,clean}$	120	96	72	48	24	0

Textbook Call policy :

$$(1-y\%)N_C\left(\mathrm{CP}_{i,clean}+F_C C_c \Delta t\right) \le CB_{i,continuation}^{total}$$

$$(1-y\%)N_C\left(\mathrm{CP}_{n,clean}+F_c C_C \Delta t\right) =$$
$$(1-60\%)\times 1\times(115+100\times 0.05\times 1) = 40\%\times 120 = 48$$

$$CB_c(x\%,y\%) =$$
$$\begin{cases} CB_{c,continuation}^{total} + (y\%-x\%)N_c F_C C_c \Delta t + (y\%-x\%)\times\gamma\times N_C\times \underline{Stock_{i,uncall}(x\%,y\%)} & ,if \quad Equity_{i,uncall}(x\%,y\%)>0 \\ \max((1-\rho)\times(V_i+\delta_i)-PV\_SBCF \quad , \quad 0) & , \quad Otherwise \end{cases}$$

$$CB_{c,continuation}^{total} = \left(p\times \underline{CB_F(60\%,100\%)} + (1-p)\times CB_F(60\%,100\%)\right)\times e^{-r_f \Delta t} + (1-y\%)\times N_C\times F_C\times C_C\Delta t$$

$$32.264 = (0.4189\times 70.0841 + (0.5811)\times 4.773)\times e^{-0.06\times 1} + (1-60\%)\times 1\times 100\times 0.05\times 1$$

$$CB_c(20\%,60\%) = 32.264 + (60\%-20\%)\times 1\times 100\times 0.05\times 1 + (60\%-20\%)\times 1.5\times 1\times 26.938 = 50.427$$

Numerical method example：C Competitive, Treasury

Competitive						
CB Value	0%	20%	40%	60%	80%	100%
0%	49.385	51.601	52.168	51.492	49.461	48.314
20%		44.307	46.659	46.867	45.226	43.812
40%			37.966	40.200	39.479	37.953
60%				29.652	31.171	29.971
80%					17.998	18.398
100%						0.000

Competitive						
Y%	0%	20%	40%	60%	80%	100%
$CB^{total}_{i,continuation}$	49.385	44.307	37.966	29.652	17.998	0
$CP_{n,clean}$	120	96	72	48	24	0

The converted value	0%	20%	40%	60%	80%	100%
0%	39.9157	36.4726	35.5046	36.4002	39.3283	48.3136
20%		47.1144	43.4672	43.0377	45.3801	43.812
40%			56.2074	52.7387	53.7014	63.2547
60%				68.2603	65.8632	74.9275
80%					85.3222	91.9877
100%						119.2840
Unconverted value	49.385	55.384	63.277	74.130	89.991	#
	0%	20%	40%	60%	80%	100%
0%	0	0	0	0	0	0
20%		0	0	0	0	0
40%			0	0	0	0
60%				0	0	0
80%					0	0
100%						0

$$\gamma \times Stock_C(x\%, y\%) + F_c C_C \Delta t \geq \frac{CB^{total}_{i,continuation}}{(1-y\%)}$$

Textbook Call policy :

$$(1-y\%) N_C \left(CP_{i,clean} + F_C C_c \Delta t\right) \leq CB^{total}_{i,continuation}$$

Model restrictions

- Debt to Equity (Monopoly case)
  - Parameter setting
  - Negotiation
- Constant interest rate

THANK YOU

Any Question?

FOR YOUR ATTENTION